Language and learning mathematics:

A sociocultural approach to academic literacy in mathematics

JUDIT N. MOSCHKOVICH, PROFESSOR, UNIVERSITY OF CALIFORNIA SANTA CRUZ, USA

This article summarizes my work on language and learning mathematics and a framework for academic literacy in mathematics. This framework can be used to analyze students' oral or written contributions or to review, design, or supplement mathematical tasks for attention to language.

Introduction

This paper summarizes a theoretical framework for *academic literacy in mathematics* (Moschkovich, 2015a; 2015b) that uses a complex view of both mathematics and language, focuses on understanding (not computation), and emphasizes mathematical practices (Moschkovich, 2013a), such as communicating mathematically. To support all students in learning mathematics we need to shift from simplified views of mathematical language as single words to a broader definition of academic literacy – not just learning words but learning to communicate mathematically. This shift to an expanded view of *academic literacy in mathematics* that integrates mathematical proficiency, practices, and discourse is crucial for all students but essential for students learning the language of instruction (Moschkovich, 2013b). Research and policy have called for mathematics instruction for these students to maintain high standards (AERA, 2004) and high cognitive demand (AERA, 2006). To accomplish these goals, mathematics instruction must shift from focusing on low-level language skills (i.e., vocabulary or single words) or mathematical skills (i.e., arithmetic computation) to using an expanded definition of *academic literacy in mathematics* that includes mathematical practices and discourse. This sociocultural framework can be used to consider how hybrid language practices provide resources for mathematical activity framed as sociocultural, not purely individual or cognitive, and to design lessons that support students in communicating and participating in order to learn mathematics.

> Mathematics instruction must shift from focusing on low-level language skills (i.e., vocabulary or single words) or mathematical skills (i.e., arithmetic computation) to using an expanded definition of *academic literacy in mathematics* that includes mathematical practices and discourse.

What is a sociocultural approach to academic literacy in mathematics?

The sociocultural framework provides an integrated view of *academic literacy in mathematics1* (Moschkovich, 2015a; 2015b). The framework draws on situated perspectives of learning mathematics as a discursive activity (Forman, 1996) that involves participating in a community of practice, developing classroom socio-mathematical norms, and using multiple material, linguistic, and social resources. Mathematical activity is assumed to involve not only individual mathematical knowledge but also collective mathematical practices and discourses. A sociocultural perspective brings several assumptions to defining academic literacy in mathematics. The first assumption is that mathematical activity is simultaneously cognitive, social, and cultural. Second, the focus is on the potential for progress in what learners say and do, not on learner deficiencies. The focus shifts from looking for deficits to identifying the mathematical discourse practices evident in student contributions (e.g., Moschkovich, 1999). The third assumption is that participants bring multiple perspectives to a situation, representations and utterances have multiple meanings, and meanings for words are situated, constructed while participating in practices, and negotiated through interaction (Moschkovich, 2002; 2004; 2007b).

What is academic literacy in mathematics?

Academic literacy in mathematics includes three integrated components: mathematical proficiency, mathematical practices, and mathematical discourse. *Academic literacy in mathematics* is more complex than simply combining alphabetic literacy with proficiency in arithmetic computation. For example, reading and solving a word problem entails not only proficiency in mathematics but also competencies in using *mathematical practices* and *discourses*. These three components are intertwined, should not be separated during instruction, and cannot be separated when analyzing student mathematical activity or designing mathematics tasks or lessons.

The view of academic literacy in mathematics presented here is different than previous approaches to academic language in several ways. First, the definition includes not only cognitive aspects of mathematical activity – what happens in one's mind, such as mathematical reasoning, thinking, concepts, and metacognition – but also social and cultural aspects – what happens with other people, such as participation in mathematical practices – and discourse aspects – what happens when using language (reading, writing, listening, or talking about mathematics). Most importantly, the components of academic literacy in mathematics work together, not separating mathematical language from proficiency or practices.

This definition goes beyond narrow views of mathematical language that limit learners' access to high-quality curriculum or instruction: a) A focus on single words or vocabulary limits access to complex texts and high-level mathematical ideas and to opportunities for students to understand and make sense of those texts, b) The assumption that meanings are static and given by definitions limits students' opportunities to make sense of mathematics texts for themselves, and c) The assumption that mathematical ideas should always and only be communicated using formal language limits the resources (including informal, everyday, or home language) that students can use to communicate mathematically.

In contrast, the view of mathematical language used here assumes that meanings for academic language are situated and grounded in the mathematical activity that students are actively en-

1 The perspective and definition of academic literacy in mathematics used here build on analyses in previous publications (Moschkovich, 2002; 2007a; 2008; 2013a).

gaged in. For example, meanings for the words in a word problem come not from a definition in a word list provided by the teacher; instead, students develop meanings as they work on a problem, communicate about a word problem with peers, and develop their solutions. A complex view of mathematical language also means that lessons must include multiple modes (not only reading and talking but also listening and writing), multiple representations (gestures, objects, drawings, tables, graphs, symbols, etc.), and multiple ways of using language (formal school mathematical language, home languages, and everyday language). In addition, this definition expands academic literacy in mathematics beyond simplified views of mathematics as computation. First, it includes the full spectrum of mathematical proficiency, balancing procedural fluency with conceptual understanding. Second, it includes mathematical practices. And lastly, it emphasizes student participation in discourse practices.

" In contrast, the view of mathematical language used here assumes that meanings for academic language are situated and grounded in the mathematical activity that students are actively engaged in.

Why is conceptual understanding important to academic literacy in mathematics?

Procedural fluency and conceptual understanding are two important strands of mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001). Fluency in mathematical procedures is what many imagine when we say „learning mathematics". Conceptual understanding, in contrast, involves the connections, reasoning, and meaning that learners (not teachers) construct; it is much more than performing a procedure accurately and quickly (or memorizing a definition or theorem); it involves understanding why a particular result is the correct answer and what that result means, i.e., what the number, solution, or result represents. For example, explaining (or showing using a picture) why the result of multiplying 1/2 by 2/3 is smaller than 1/2.

Another aspect of conceptual understanding is connecting representations (i.e., words, drawings, symbols, diagrams, tables, graphs, equations, etc.), procedures, and concepts (Hiebert & Carpenter, 1992). For example, if a student understands addition and multiplication, we expect that they made connections between these two procedures, and that they could explain how multiplication and addition are related (i.e., that multiplication can be repeated addition). If they understand the procedures for multiplying and dividing negative numbers, we expect that they made connections between these two procedures and that they could explain how the procedures for multiplication and division are similar, different and explain why.

Even though procedural fluency matters, if we want students to learn and remember procedures (i.e., multiplication facts or procedures for dividing fractions), conceptual understanding is crucial. Conceptual understanding and procedural fluency are closely related, even if we, as adults, do not remember understanding a particular procedure when we learned it. Research in cognitive science (Bransford, Brown, & Cocking, 2000) has shown that people remember better, longer, and in more detail if they understand, elaborate, actively organize, and connect new knowledge to prior knowledge. Thus, children will remember procedures better, longer, and in more detail if they actively make sense of procedures, connect procedures to other procedures, and connect procedures to concepts and representations. Rehearsal (repeating something over and over) may work for memorizing a grocery list (even then, organizing the list improves memorization). Rehearsal, however, is not the most efficient strategy for remembering how to perform demanding cognitive tasks, such as arithmetic operations. The research evidence is clear. The best way to remember is to understand, elaborate, and organize what you know (Bransford, Brown, & Cocking, 2000). One way we elaborate is by talking or writing about our mathematical thinking and hearing or seeing others' solutions, making mathematical discourse crucial for understanding.

> **The best way to remember is to understand, elaborate, and organize what you know.**

What are mathematical practices and discourse?

Mathematical proficiency (Kirkpatrick et al., 2001) provides a cognitive account of mathematical activity focused on knowledge, metacognition, and beliefs. A sociocultural perspective adds participation in mathematical practices (Moschkovich, 2013), such as problem-solving, sensemaking, reasoning, modeling, and looking for patterns, structure, or regularity. The term *practices* shifts from purely cognitive accounts of mathematical activity to assuming the social, cultural, and discursive nature of doing mathematics. I use the terms *practices* drawing on Scribner's (1984, p. 13) practice account of literacy to „highlight the culturally organized nature of significant literacy activities and their conceptual kinship to other culturally organized activities involving different technologies and symbol systems". This definition implies that mathematical practices are culturally organized, involve symbol systems, and are related conceptually to other mathematical practices. From this perspective, mathematical practices are not only cognitive – i.e., involving mathematical thinking and reasoning – but also social and cultural – arising from communities and mark membership in communities – and semiotic – involving semiotic systems (signs, tools, and their meanings).

A sociocultural framing of mathematical practices connects practices to discourse. In particular, discourse is central to participation in many mathematical practices, and meanings for words are situated and constructed while participating in mathematical practices.[2] I use the phrase *mathematical discourse* (Moschkovich, 2007a) to mean the *communicative competence* (Hymes, 1972/2009) necessary and sufficient for competent participation in mathematical practices and to emphasize that discourse is much more than language.

Academic mathematical discourse has been described as having some general characteristics. In general, particular modes of argument, such as precision, brevity, and logical coherence, are valued (Forman, 1996). Abstracting, generalizing, and searching for certainty are also highly valued. Generalizing is reflected in common mathematical statements, such as „The angles of any triangle add up to 180 degrees", „Parallel lines never meet", or „a + b (always) equals b + a". What makes a claim mathematical is, in part, the detail in describing when the claim applies and when it does not. Mathematical claims apply only to a precisely and explicitly defined set of situations and are often tied to mathematical representations (symbols, graphs, tables, or diagrams).

2 I am putting aside the relationship between mathematical practices and discourse, including questions regarding whether all mathematical practices are discursive, whether some are more discursive than others, etc. These complex issues are discussed elsewhere (Moschkovich, 2013a).

> Academic mathematical discourse has been described as having some general characteristics. In general, particular modes of argument, such as precision, brevity, and logical coherence, are valued.

Why is discourse important for learning mathematics?

Communication is important because it supports conceptual understanding. The more opportunities a learner has to make connections among multiple representations, the more opportunities that learner has to develop conceptual understanding. But not all kinds of communication support conceptual understanding in mathematics. Communication must be focused on important mathematical ideas. Classroom communication that engages students in evidence-based arguments by focusing on explanations, arguments, and justifications builds conceptual understanding. Communication should also include multiple modes (talking, listening, writing, drawing, etc.), because making connections among multiple ways of representing mathematical concepts is central to developing conceptual understanding.

How can instruction focus on academic literacy in mathematics?

Mathematics lessons that pay attention to language need to include the full spectrum of mathematical proficiency (balancing computational fluency with tasks that require conceptual understanding), provide opportunities to participate in mathematical practices, and include multiple discourses as resources (Moschkovich, 2013a; 2013b). Instruction should allow students to use multiple resources (i.e., modes of communication, symbol systems, or languages) for mathematical reasoning (Moschkovich, 2014a; 2014b) and support students in negotiating meanings for mathematical language grounded in student mathematical work instead of giving students definitions separate from mathematical activity (Moschkovich, 2015a; 2015b).

Guidelines for mathematics instruction that pays attention to language include:

- Support student participation in mathematical discussions (for examples, see Moschkovich, 1999; 2002; 2007a; 2007b).
- Focus on mathematical practices, such as reasoning and justifying, not vocabulary or accuracy in using individual words (for examples, see Moschkovich 1999; 2002; 2007a; 2007b).
- Treat everyday and home languages as resources, not deficits (see Moschkovich 2002).
- Draw on multiple resources available in classrooms – objects, drawings, graphs, and gestures – as well as home languages and experiences outside of school.

The question is not whether students should learn vocabulary but rather when and how instruction can best support students as they learn not only the meanings of words and phrases but also how to participate in mathematical practices and discourse. Vocabulary drill, practice, definitions, or lists are not the most effective way to learn to communicate mathematically. Instead, vocabulary acquisition (in a first or second language) occurs most successfully in instructional contexts that are language-rich, actively involve students in using language, require both receptive and expressive understanding, and require students to use words in multiple ways over extended periods of time (Blachowicz & Fisher, 2000; Pressley, 2000). To develop oral and written communication, students need to participate in negotiating meanings (Savignon, 1991) and in tasks that require student output (Swain, 2001). Instruction should provide opportunities to actively use mathematical language to communicate about and negotiate meaning for mathematical situations.

Separating language from mathematical proficiency and focusing on words, vocabulary, or definitions, limits learners' access to conceptual understanding. Separating language from mathematical practices curtails students' opportunities to participate in mathematical practices. Not allowing students to use informal language, typically acquired before more formal ways of talking, also limits the resources to communicate mathematically. Lastly, focusing on correct vocabulary curtails opportunities for students to express themselves mathematically in what are likely to be imperfect ways, especially as they are learning new ideas. In contrast, the view of academic literacy in mathematics described here provides a complex and expanded view of mathematical language that starts with conceptual understanding, focuses on mathematical practices and discourse, and includes informal language as a resource.

To summarize, mathematics instruction needs to support students both to reason mathematically and to express that reasoning. However, for students learning mathematics, informal language is important, especially when students are exploring a new mathematical concept or discussing a math problem in small groups. Students can use informal language during exploratory talk (Barnes, 2008) or when working in a small group (Herbel-Eisenmann et al., 2013). Such informal language reflects important mathematical thinking (for examples, see Moschkovich, 1996; 2008). In other situations, for example, when presenting a solution or writing an account of a solution, using more formal academic mathematical language becomes more important.

Recommendations for research and practice

Teachers can choose (or design) tasks that support academic literacy in mathematics, provide opportunities for participation in academic literacy in mathematics, and recognize academic literacy in mathematics in student activity. Teachers can consider each component and how to provide opportunities for participation in each of the components. Below are some questions to ask when selecting or adapting math tasks that pay attention to language:

- **Conceptual Understanding:** Is conceptual understanding necessary or possible with the task? Can the task be modified to include conceptual understanding?
- **Math Practices:** Which math practices are necessary or possible for solving the problem? Are additional math practices possible? What participation structures are necessary to engage students in those math practices?
- **Math discourse:** What typical math discourse modes (listening, talking, reading, or writing) are involved or possible? What purposes and representations are involved or possible? Are there any language resources that are specific to these students or their community?

We must leave behind simplified views of language as vocabulary, embrace the multimodal and multi-semiotic nature of mathematical activity, and shift from monolithic views of math talk or dichotomized views of everyday and mathematics registers (Moschkovich, 2010). An overemphasis on correct vocabulary and formal language limits the linguistic resources teachers and students can use in the classroom to learn mathematics with understanding. Work on the language of disciplines provides a complex view of mathematical language as not only specialized vocabulary – new words and new meanings for familiar words – but also as extended discourse that includes syntax, organization, the mathematics register, and discourse practices.

> We must leave behind simplified views of language as vocabulary, embrace the multimodal and multi-semiotic nature of mathematical activity, and shift from monolithic views of math talk or dichotomized views of everyday and mathematics registers.

Overall, research and practice need to avoid dichotomies such as everyday/academic or formal/informal (Moschkovich, 2010). Classroom discussions draw on hybrid resources from both academic and everyday contexts, and multiple registers co-exist in math classrooms. Most importantly for supporting the success of students who are learning the language of instruction, mathematical discussions need to build on the language students bring from their communities. Therefore, everyday ways of talking should not be seen as obstacles to participation in academic mathematical discussions but as resources teachers can build on to support students in learning more formal mathematical ways of talking. Teachers need to hear the mathematical content in students' everyday language, build on that everyday language, and support or scaffold (Moschkovich, 2015c) more formal language. Everyday language is not only a starting place for learners. Everyday and home languages facilitate communication, ground meaning, and honor students' home language practices (e.g., norms for turn-taking, interrupting, or showing respect).

References

American Educational Research Association (2004). Closing the gap: High achievement for students of color. *Research Points, 2*(3).

American Educational Research Association (2006). Do the math: Cognitive demand makes a difference. *Research Points, 4*(2).

Barnes, D. (2008). Exploratory talk for learning. In N. Mercer & S. Hodgkinson (Eds.), *Exploring talk in school* (pp. 1-15).

Blachowicz, C., & Fisher, P. (2000). Vocabulary instruction. In M. L. Kamil, P. B. Mosenthal, P. D. Pearson, & R. Barr (Eds.), *Handbook of Reading Research*, V 3, pp. 545-561. Erlbaum.

Bransford, J., Brown, A., & Cocking, R. (2000). *How people learn: Brain, mind, experience, and school.* National Academy Press.

Forman, E. (1996) Learning mathematics as participation in classroom practice: Implications of sociocultural theory for educational reform. In L. Steffe, P. Nesher, P. Cobb, G. Goldin, & B. Greer (Eds.), *Theories of mathematical learning*. Erlbaum.

Herbel-Eisenmann, B. A., Steele, M. D., & Cirillo, M. (2013). Developing teacher discourse moves: A framework for professional development. *Mathematics Teacher Educator*, *1*(2), 181-196.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-97). Macmillan.

Hymes, D. (2009). On communicative competence. In A. Duranti (Ed.), *Linguistic anthropology: A reader* (Vol. 1). John Wiley & Sons. (Reprinted from *Sociolinguistics,* pp. 269-293, by J. B. Pride & J. Holmes, Eds., 1972, Penguin.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. National Research Council. National Academy Press.

Moschkovich, J. N. (1996). Moving up and getting steeper: Negotiating shared descriptions of linear graphs. *Journal of the Learning Sciences*, *5*(3), 239-277.

Moschkovich, J. N. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics*, *19*(1), 11-19.

Moschkovich, J. N. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning*, *4*(2&3), 189-212.

Moschkovich, J. N. (2007a). Examining mathematical discourse practices. *For The Learning of Mathematics*, *27*(1), 24-30.

Moschkovich, J. N. (2007b). Bilingual mathematics learners: How views of language, bilingual learners, and mathematical communication impact instruction. In N. Nasir & P. Cobb (Eds.), *Diversity, equity, and access to mathematical ideas* (pp. 89-104). Teachers College Press.

Moschkovich, J. N. (2008). „I went by twos, he went by one“: Multiple interpretations of inscriptions as resources for mathematical discussions. *Journal of the Learning Sciences*, *17*(4), 551-587.

Moschkovich, J. N. (2010). Language(s) and learning mathematics: Resources, challenges, and issues for research. In J. N. Moschkovich (Ed.), *Language and mathematics education: Multiple perspectives and directions for research* (pp. 1-28). Information Age Publishing.

Moschkovich, J. N. (2013a). Issues regarding the concept of mathematical practices. In Y. Li & J. N. Moschkovich (Eds.), *Proficiency and beliefs in learning and teaching mathematics: Learning from Alan Schoenfeld and Günter Toerner* (pp. 257-275). Sense Publishers.

Moschkovich, J. N. (2013b). Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education, 6*(1), 45-57.

Moschkovich, J. N. (2014a). Building on student language resources during classroom discussions. In M. Civil & E. Turner (Eds.), *The Common Core State Standards in mathematics for English language learners: Grades K-8* (pp. 7-19). TESOL International Association.

Moschkovich, J. N. (2014b). Language resources for communicating mathematically: Treating home and everyday language as resources. In T. Bartell & A. Flores (Eds.), *Embracing resources of children, families, communities and cultures in mathematics learning*. TODOS Research Monograph (Vol. 3, pp. 1-12). Create Space Independent Publishing Platform.

Moschkovich, J. N. (2015a). Academic literacy in mathematics for English Learners. *Journal of Mathematical Behavior,* 40, 43-62.

Moschkovich, J. N. (2015b). A sociocultural approach to academic literacy in mathematics for adolescent English Learners: Integrating mathematical proficiency, practices, and discourse. In D. Molle, E. Sato, T. Boals, & C. Hedgspeth (Eds.), *Multilingual learners and academic literacies: Sociocultural contexts of literacy development in adolescents* (pp. 75-104). Routledge.

Moschkovich, J. N. (2015c). Scaffolding mathematical practices. *ZDM, 47*(7), 1067-1078.

Pressley, M. (2000). What should comprehension instruction be the instruction of? In M. L. Kamil, P. B. Mosenthal, P. D. Pearson, & R. Barr (Eds.), *Handbook of reading research* (Vol. 3, pp. 545-561). Erlbaum.

Savignon, S. (1991). Communicative language teaching. *TESOL Quarterly*, 25(2), 261-277.

Scribner, S. (1984). Studying working intelligence. In B. Rogoff & J. Lave (Eds.), *Everyday cognition: Its development in social context* (pp. 9-40). Harvard University Press.

Swain, M. (2001). Integrating language and content teaching through collaborative tasks. *Canadian Modern Language Review*, *58*(1), 44-63.

About the author

Judit Moschkovich is currently Professor of Mathematics Education in the Education Department at the University of California Santa Cruz. She uses sociocultural approaches to study mathematical thinking and learning in three areas: algebraic thinking, mathematical discourse, and mathematics learners who are bilingual, learning English, and/or Latinx.